NONLINEAR ULTRASOUND PROPAGATION MODELING

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ABSTRACT

As commonly applied intensities of ultrasound grow there is a concern that linear assumptions in ultrasound propagation modeling are not sufficient and nonlinear effects have to be taken into account. Presented paper deals with a numerical model implementation allowing to simulate nonlinear ultrasound propagation in physical media. Finally, simulation results of linear and nonlinear models are compared for a set of input acoustic pressure amplitudes.

1. INTRODUCTION

Numerical modeling of ultrasonic transducers pressure field plays an important role in setting safety precautions for both diagnostic and therapeutic ultrasound applications. Currently both linear and nonlinear numerical models are used. The paper focuses on description of a nonlinear model and discuses the conditions under which using this model instead of the linear one is necessary.

2. THEORETICAL BACKGROUND

Propagation of ultrasound in a real media is influenced by the fact that the local propagating speed value is dependant on the local value of the acoustic pressure, which causes nonlinear distortion to the traveling wave. The dependency can be described by equation (1), [3]:

$$c = c_0 \left[1 + \frac{B}{2A} \frac{u}{c_0} \right]^{(2A/B)+1} \qquad u = p / Z_0 \tag{1}$$

Where c_0 is the ambient propagating speed of ultrasound, B/A is the parameter of non-linearity of the media, Z_0 is the acoustic impedance of the media and u is the acoustic speed dependant on the acoustic pressure p.

Including equation (1) into our computations leads to several important implications: The propagating speed of ultrasound in a homogenous physical media is not constant. Especially at higher amplitudes when dependency of propagating speed on acoustic pressure is too strong, the principle of superposition cannot be used. As will be shown further, increasing input amplitude by a certain factor for example does not cause an increase in amplitude by exactly the same factor in the other places of the medium.

Due to computing speed reasons, linear numerical models of ultrasound propagation are often used, where equation (1) is omitted and propagating speed c is treated as a constant equal to c_0. This approach provides reasonably accurate results at lower amplitudes but limitations in accuracy appear in using linear models with higher acoustic pressure amplitudes.

3. NONLINEAR SIMULATION ALGORITHM

An algorithm based on finite difference time domain (FDTD) method was developed in Matlab environment. As seen in fig. 1, besides usual numerical solution to 3D wave equation, it computes actual local propagating speed for all simulated nodes of the environment in each time sub step. This allows simulating nonlinear propagation of the ultrasound at higher amplitudes.

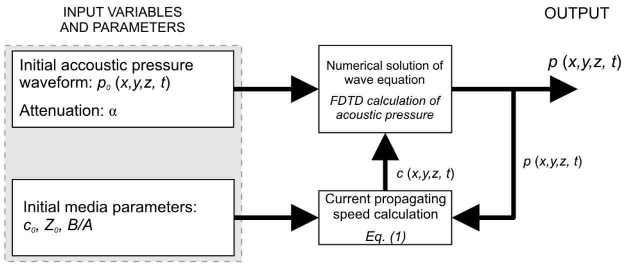

Fig. 1: A block diagram of the algorithm

Usually the main interest is in the focal area of the transducer or in its axis and the wave propagation in the other parts of the medium does not need to be simulated. To reduce the computing overhead in such situation, whole simulation task can be reduced only to the actual area of interest as described in fig. 2.

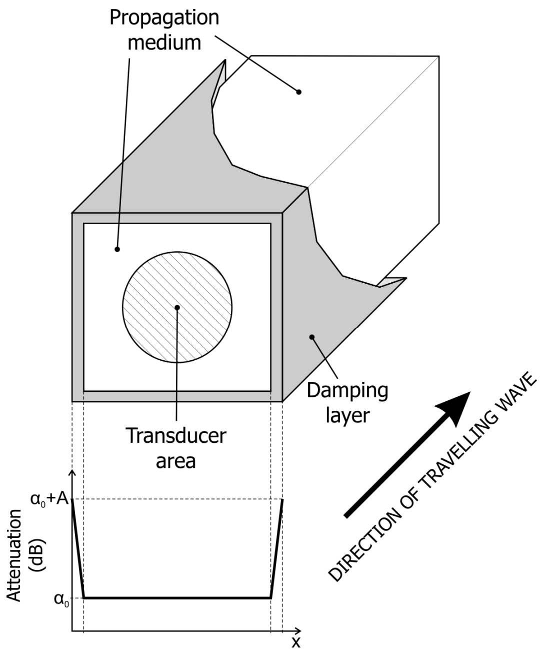

Fig. 2: Simulation task layout. Damping layer helps to reduce the volume of simulated medium only to the actual area of interest.

The area of interest is encapsulated by a thin damping layer which purpose is to eliminate any unwanted reflections on the boundaries of the medium. The parameters of the damping layer are all the same as the parameters of the simulated medium except for the attenuation α which magnitude gradually increases with distance from the transducer as seen in the graph in fig. 2.

4. SIMULATION RESULTS

A comparison of linear and nonlinear model is made in fig. 3 and 4. Acoustic pressure along the axis of a circular transducer of 5 mm in diameter was computed for a set of input pressure amplitude values. Simulation parameters are: f_0 = 2 MHz; B/A = 5.5; α = 4.5 Np/m; c_0 = 1600 m/s; Z_0 = 1.5 MPa.s.m^{-1}.

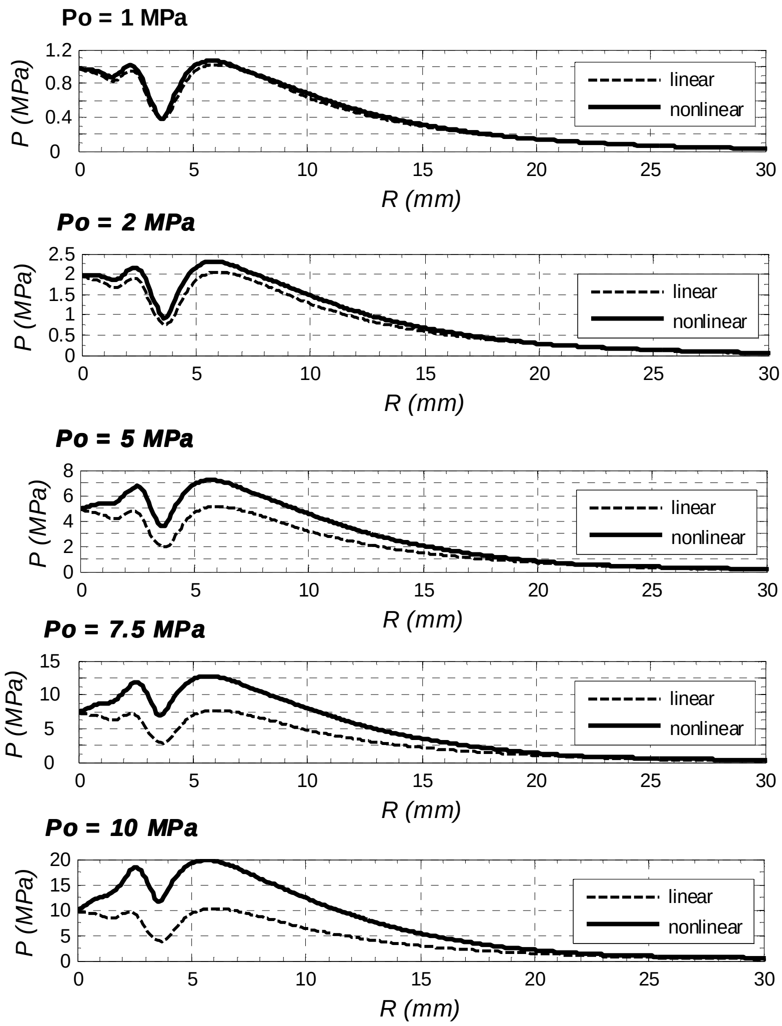

Fig. 3: Comparison of simulation results between linear and non-linear propagation model in computing acoustic pressure amplitude along the transducer axis.

Results show that difference in values obtained by linear and nonlinear propagation model grows with applied input acoustic pressure amplitude.

Figure 3 shows the comparison of acoustic pressure amplitude along the transducer axis computed by linear and nonlinear model.

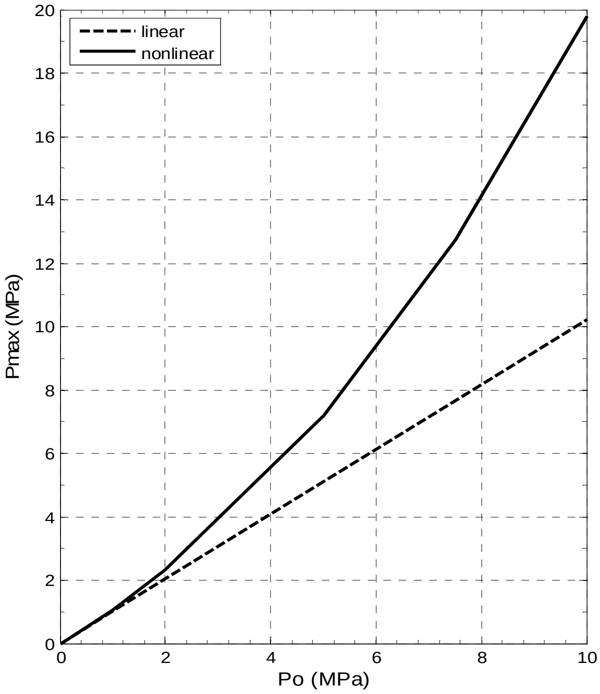

Fig. 4: The amplitude in the last maximum of the transducer axis computed by linear and nonlinear propagation model.

Figure 4 shows how the amplitude observed in the last maximum on the transducer axis changes with the input amplitude at the transducer surface. The linear model expects the superposition principle to be valid for all amplitudes and therefore the computed amplitude in the maximum is a linear function of the input amplitude. Using nonlinear model we get a nearly linear dependency for amplitudes below 1 MPa in our particular example. Above this level, a nonlinear dependency can be seen.

5. DISCUSSION

Using a linear model for high amplitudes leads to underestimating the pressure field. The exact value of amplitude threshold, where switching to nonlinear model is necessary, depends especially on *B/A*, Z_0 and c_0 values and also on the type of the actual simulation task and desired accuracy. Simulation results support conclusions based on a different nonlinear numeric model presented in [2] and agree with theory and experimental results published in [1].

Input acoustic pressure amplitudes of present CW ultrasound diagnostic instruments are commonly below 0.1 MPa to meet World Federation of Ultrasound in Medicine and Biology safety limit of 720 $mW.cm^{-2}$ intensity for continuous exposition. For these amplitudes non-linear modeling is not necessary. However in pulse mode instruments like pulsed Doppler or color Doppler, peak amplitudes may reach much higher values. For example, peak input amplitudes of up to 5 MPa can be produced by color Doppler instruments [4]. In such cases non-linear models can provide improved accuracy in pressure field prediction.

6. CONCLUSION

An algorithm for simulating nonlinear ultrasound propagation was designed. Simulation results were compared to the output of a simplified linear model. The comparison shows that nonlinear model is necessary for achieving accurate results at high input amplitudes. However at lower amplitudes, using linear propagation model may not cause any major problems and benefits of improved computing speed can be taken. The threshold for switching between the linear and nonlinear propagation models depends on the actual simulation task and parameters of the propagation medium.

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